



Resonant Electro-Mechanical Coupling – A Possible Renewable Energy Source

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Abstract

This paper looks at a simple Electro-Mechanical System that is tuned to resonate with both mechanical and electrical resonance. The intention is to couple the system resonance such that the amplitude of the output voltage is increased. The basic setup consists of a forced Mass-Spring System consisting of a speaker driving a mass attached to a tension-spring. The mass is a Neodymium cylindrical magnet. The vertically driven system couples to a magnet coil which sits over the magnet. Thus, by induction a voltage is generated within the coil. The coil is connected in series with a set of electrolytic capacitors. The resulting Resistor-Capacitor-Inductor (RCL) circuit is so designed as to resonate. The idea behind this coupling is to increase the output voltage of the circuit, thereby creating a voltage supply that could be boosted via transformer action, if necessary. Noting that the resulting circuit provides a source of single-phase electricity, such a setup could be used to charge small electronic devices like cellphones and mobile earphone sets. Mechanical resonance is indicated by the erratic vibration of the speaker, spring and magnet system. Knowing the magnet mass and measured resonant frequency the approximate spring constant can be calculated. The resonant frequency is then used as the driving frequency for the electrical circuit consisting of the RCL components. Noting the electric circuit resonance frequency, the approximate capacitance needed can be calculated using the driving frequency and the inductance of the said magnet coil. Using the circuit capacitance and appropriate circuit resistance, the maximised quality factor is sought, therefore the objective of the electric circuit design is to maximise the output voltage. The power generated by the device was found to be less than ~1 W. This system can be optimized to deliver maximum power output at the required resonant frequency, by greatly increasing the coil turns, using closer tolerances between the moving magnet and the wound coil and/or using materials that would ensure added magnetic coupling. Such a system can be used as a possible source of renewable energy especially on reciprocating engines, where exciting forces are available.

Keywords: *Reliability, availability, renewable energy, alternate*

Introduction

This paper looks at a simple Electro-Mechanical System that is tuned to resonate with both mechanical and electrical resonance (Hasan, et al., 2018; Tesla Research, 2023). The principles of operation are similar to that of an “eddy current tube” (Hosseini & Morris, 2006; Kraftmakher, 2000), where the tube is replaced by a magnet coil. The intention is to couple the system resonance so that the amplitude of the output voltage is increased. The basic setup consists of a forced Mass-Spring System consisting of a speaker driving a mass attached to a tension-spring. The mass is a Neodymium cylindrical magnet. The vertically driven system couples to a magnet coil which moves over the magnet. Thus, by induction a voltage is generated within the coil. The coil is connected in series with electrolytic capacitors. The resulting Resistor-Capacitor-Inductor- (RCL) circuit is so designed as to resonate and operate at minimum impedance (Hasan et.al., 2018). The idea behind this coupling is to increase the

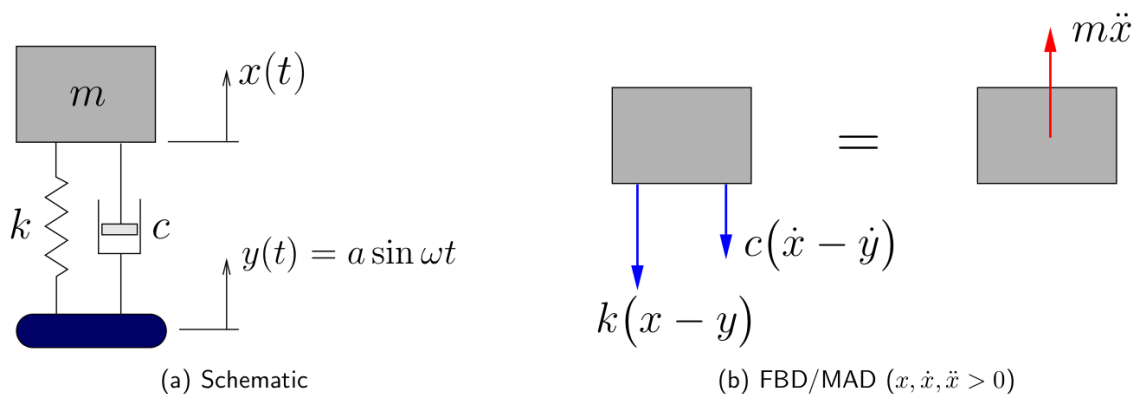
output voltage of the circuit, thereby creating a voltage supply that could be possibly boosted via transformer action, if necessary. Energy harvesting which implies “the generation of electrical power from the kinetic energy available in the environment” (Gieras, Oh, & Huzmezan, 2008) is an active field of research (Mohanty, et al., 2019; Muscat, et al., 2022; Many of the advances in this field are in the area of micro-devices (Caldwell, 2016), whereas, the device presented here measures about 100mm in height (Chiu, et al., 2016).

Mass Spring Damper System with Base Excitation

A spring-mass-damper system can be modelled as in Figure 1(a), below (UofA, 2023). The free-body diagram is shown in Figure 1(b) (UofA, 2023), where the balance of forces is shown. Here, $y(t)$ depicts the upward movement of the base (base excitation), $x(t)$ depicts the upward displacement of the mass (m). Also, k is the spring-constant of the system, while c represents the system damping.

Figure 1

Spring Mass Damper System Model



Equation 1, is the equation of motion of the spring-mass-damped system with forced harmonic base excitation (UofA, 2023) (Inman, 2001). Here, a is the amplitude of the excitation, while ω is the frequency of base excitation.

$$m\ddot{x} + c\dot{x} + kx = ka \sin (\omega t) + ca\omega \cos (\omega t) \quad \text{Equation (1)}$$

Equation (2), represents the natural frequency of the system, where the effects of damping are ignored and the system is deemed to behave purely linearly (Anderson, 1967) (Meirovitch, 1986). It can be concluded that the system natural frequency depends primarily on the mass and spring constant of the system (Inman, 2001) (Timoshenko, Young & Weaver, 1974).

$$\omega_r = \sqrt{k/m} \quad \text{Equation (2)}$$

The mass-spring system with forced base excitation, discussed above represents a model of the audio speaker, spring, Neodymium magnet, physical setup. At resonance the system behaves erratically tending towards maximum displacement (French, 1996). The frequency at which this maximum displacement happens, is noted and recorded as the system natural frequency. When testing the said system (see Fig. 1), the recorded natural frequency was about 47 Hz. Using Equation (2), the estimated spring constant was calculated as 1 570 N/m, using the magnet mass of 18 g (see Table 1). The forced harmonic excitation is a square-wave with 10 V peak-to-peak.

Figure 2*Spring Mass System with Base Excitation***Table 1***Spring Mass System Parameters*

Component	Description - Specifications
Audio Speaker	Power = 5W Impedance = 8Ω
Magnet	NeFeB Mass = 18 g
Spring	Steel Length = 30 mm Diameter = 5 mm
Function Generator	Square Wave 10V p-p Frequency = 47 Hz

1. Application of Faraday's Law of Electro-Magnetic Induction and Lenz's Law

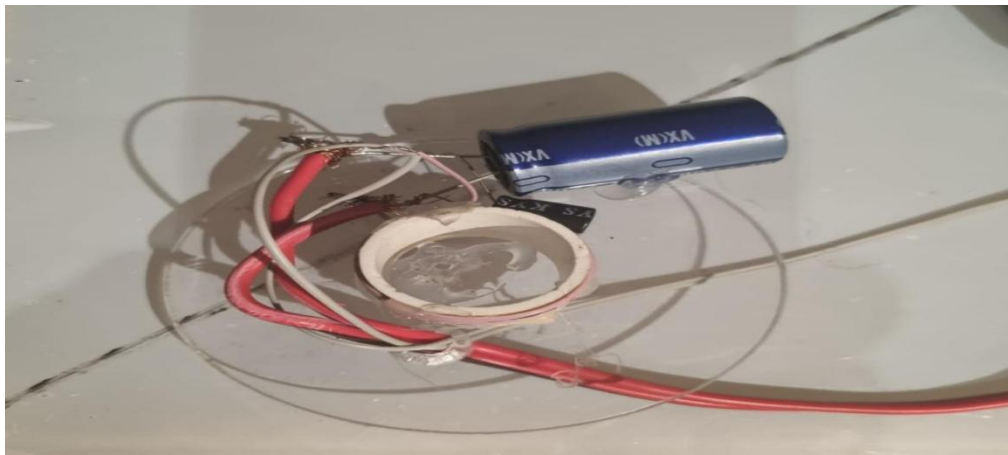
Equation (3), shows Faraday's Law (Nasar, 1981; Griffiths, 1999), where the voltage (electromotive force) (v) is generated by the change in magnetic flux $d\phi/dt$ passing through a stationary cylindrical coil of wire, the flux is generated by movement of a cylindrical magnet through the coil. In Equation 3, N represents the number of turns of the coil. Thus, the generated voltage can be increased by increasing the number of coil turns and/or increasing the rate of change of the magnetic flux "cutting" the coil wires. Lenz's law (Griffiths, 1999; Nasar, 1981) indicates that the induced magnetic field voltage opposes the direction of the applied change in magnetic flux. This implies that a sinusoidal voltage (emf) would be generated by repeated motion of the magnet (in and out) of the coil of wire.

$$v = -N \frac{d\phi}{dt} = \text{emf (V)}$$

Equation (3)

Figure 3

Wire Coil with Connected Capacitors



2. The Series RLC Circuit at Resonance

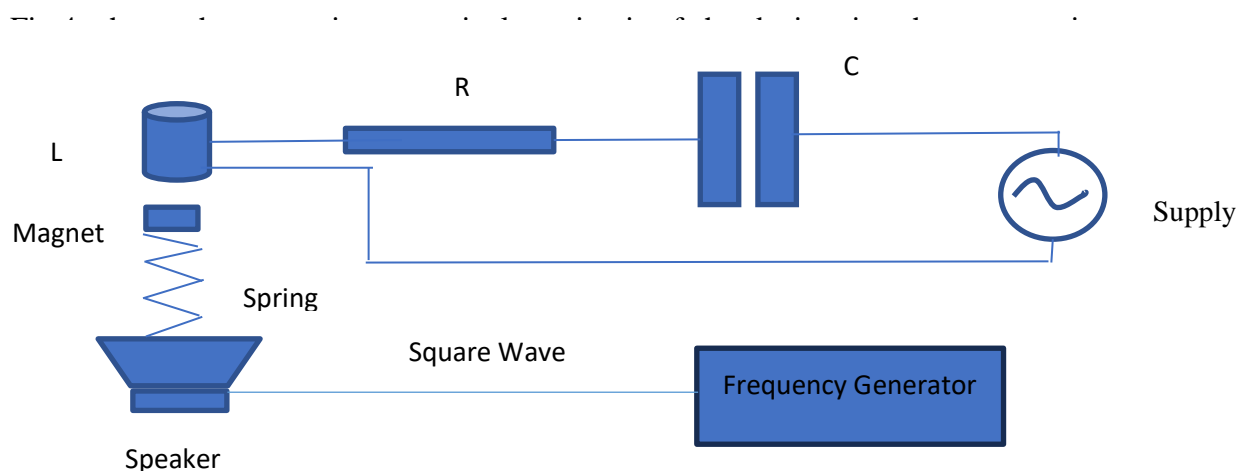
At resonance a series RLC circuit exhibits equal inductive and capacitive reactances in magnitude, which cancel each other because they are 180 degrees out of phase (Halliday, Resnick & Walker, 2003) (Hyat & Kemmerly, 1978). The sharp minimum in impedance (resistance) which occurs (at resonance) is useful in tuning applications of filter circuits (GSU, 2023) (Hyat & Kemmerly, 1978). The sharpness of the minimum depends on the value of R and is characterized by the "Q" (Quality Factor) of the circuit (GSU, 2023).

$$X_C = 1/\omega C ; X_L = \omega L ; \omega_r = 1/\sqrt{LC} \quad \text{Equation (4)}$$

Equation (4) (GSU, 2023) shows that at resonance the capacitive reactance X_C equals the inductive reactance X_L , which leads to the resonant frequency of the series RLC circuit, $\omega = 1/\sqrt{LC}$ (Hyat & Kemmerly, 1978).

Figure 4

Electro-Mechanical Resonant Circuit



voltage and current output measures at 0.593 W, 15.9 mV and 487 μ A. Test (2), shows that with the addition of circuit capacitance, the power, voltage and current output measures at 0.797 W, 19.9 mV and 584 μ A. Test (3), shows that with the addition of circuit capacitance near resonance, the power, voltage and current output measures at 0.867 W, 19.8 mV and 627 μ A. The test results show that if the circuit resistance could be minimized, the system output voltage and power could be increased.

Table 2

Tests Showing Effects of Increased Capacitance

Circuit Parameters	Test (1)	Test (2)	Test (3)
Resistance $\text{\textcircled{R}}$	0.4 Ω	0.428 Ω	0.454 Ω
Inductance (L)	0.2 mH	0.2 mH	0.2 mH
Capacitance $\text{\textcircled{C}}$	0	2300 μ F = 2200 μ f + 100 μ F	57300 μ F = 2200 μ f + 100 μ F + 55000 μ F
Voltage Out (mV)	15.9	19.9	19.8
Current Out (μ A)	487	584	627
Power Out (W)	0.593	0.797	0.867

$$P_{avg} = \frac{v_{RMS}^2}{R} \quad \text{Equation 5}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} \quad \text{Equation 6}$$

The Q-Factor (Eq. 6) (Hyat & Kemmerly, 1978) of the electric circuit appears to diminish as the capacitance is added, due to the circuit resistances that are added, when connecting the respective capacitor cables (see Table 3). The Q-Factor is relatively weak (<10) (GSU, 2023) and indicates that the circuit has relatively high energy losses. It should be noted that if the circuit were finely tuned and components mounted on a well-designed Printed Circuit Board (PCB) using surface mounted devices (SMD), the Q-Factor could be increased.

Table 3

Quality Factor Calculations

Q Factor	Test (1)	Test (2)	Test (3)
R = 0.4 Ω	0.148		
R = 0.428 Ω		0.138	
R = 0.454 Ω			0.13

Conclusion

Noting the generated power from the erratic movement of the magnet moving through the coil wire (see Table 3), it is apparent that as circuit parameters (RCL) are chosen that coincide to produce the input resonant frequency, the output power (RMS), would increase, accordingly. Although the circuit components were chosen to resonate at about 47 Hz, this value could be increased by manipulating the spring stiffness (k) and/or the vibrating mass. Although the output power (RMS) was measured at about 0.867 W (RMS) (< 1 W), this value could be increased by reducing the coil resistance and/or increasing the coil inductance. By increasing the coil inductance, a smaller electrolytic capacitor could be employed to produce the resonant frequency. It should be noted that the 55000 μ F electrolytic capacitor was relatively expensive and is commonly in Solar PV inverter circuits. For the spring mass system to work as a proposed, typically a reciprocating base is needed, very much like a flat surface which faces upwards on an (Internal Combustion) IC engine (maybe a tractor engine or standby generator engine). If the mechanical components (spring and magnet) and electrical circuit is finely tuned, an alternating current supply can be drawn to power small house hold appliances (Caldwell, 2016). It should be noted that in these tests the load impedance has not been taken into account, thus the requirement to increase the output voltage of the magnet coil is required.

Recommendations

The electro-mechanical resonant spring mass system and electrical circuit described above shows potential as a renewable energy source, if parameter values are finely tuned and engineered to make maximum use of the incumbent environment. To increase the output power of the electric circuit, the coil inductance could be greatly increased by adding many more coil turns. The magnetic coupling could be increased by using closer tolerances between the moving magnet and the wound coil and/or using materials that would ensure added magnetic coupling.

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